Int J Theor Phys (2009) 48: 3287–3292
DOI 10.1007/s10773-009-0129-4

Time Invariance as an Additional Constraint on Nonlocal Realism

Koji Nagata · Tadao Nakamura

Received: 19 June 2009 / Accepted: 17 August 2009 / Published online: 2 September 2009

Abstract We assume that time invariance of physical laws is true. We assume that one source of $2N$ uncorrelated spin-carrying particles emits them in a state, which can be described as a multipartite pure uncorrelated state ($+\infty > N \geq 1$). We assume that each of them is a spin-$1/2$ pure state lying in the $\frac{z+x}{\sqrt{2}}$ direction. We assume that the measurement setup is two-orthogonal-settings for each of the observers. We show that $2N$-particle pure uncorrelated quantum state violates a time invariant nonlocal realistic theory. $2N$ implies that we consider Bose-Einstein statistics.

Keywords Quantum nonlocality · Quantum measurement theory · Quantum computer

1 Introduction

Realistic theories are as follows [1, 2]: Realistic theories state that there exist the elements of physical reality [3] independent of whether it is observed or not. Einstein, Podolsky, and Rosen (EPR) locality condition states that space-like separated measurement outcome is mutually independent. Some quantum predictions violate Bell inequalities [4], which form necessary conditions for local realistic theories for the results of measurements. Some quantum predictions do not accept a local realistic interpretation.

Leggett-type nonlocal realistic theory [5] is experimentally investigated [6–8]. These experiments report that some quantum predictions do not accept Leggett-type nonlocal realistic interpretation. These experiments are performed by using entangled states (two spins $\frac{1}{2}$).

Rotational invariance of physical laws is an accepted principle in classical physics. It states that the value of a correlation function does not depend on the coordinate systems used by the observers. The measurement setup classifies realistic theories [7–9]. Throughout this paper, the measurement setup is two-orthogonal-settings for each of the observers when we consider nonlocal realistic theories. We do not impose rotational invariance on nonlocal realistic theories.

K. Nagata (✉) · T. Nakamura
Obihiro University of Agriculture and Veterinary Medicine, Obihiro, Japan
e-mail: ko_mi_na1@yahoo.co.jp

Time invariance of physical laws is an accepted principle in classical physics. It states that the value of a correlation function does not depend on time. Throughout this paper, we assume time invariance of physical laws is true.

The current explosive interest in the applications of finite number photons to linear optical quantum computation [10–15] prompts us to seek direct qualitative and quantitative signatures of a multi spin-$1/2$ system. Nonlocal realistic theories seem to resist implementations of quantum computation because it is questionable that the inherent parallelism of quantum computation is depictured by a classical concept, realism, even though we accept time invariance and we give up both EPR locality condition and rotational invariance. We consider the relation between a multi spin-$1/2$ system and time invariant rotationally variant nonlocal realistic theories.

Complete understanding between a multi spin-$1/2$ system and time invariant rotationally invariant local realistic theories is not reported. In a finite-dimensional space, Werner theoretically discusses that all separable states admit time invariant rotationally invariant local realistic theory [16]. The discussion does not coexist with the discussion presented in Ref. [17]. Some separable states do not admit time invariant rotationally invariant local realistic theory if all quantum observables in time invariant rotationally invariant local realistic theory must commute simultaneously [18]. As for the difference between quantum separability and time invariant rotationally variant local realistic theory, Roy derives [19] so-called 'the Roy inequality' with two-orthogonal-settings for each of the observers. Greenberger, Horne, and Zeilinger (GHZ) state [20] violates the Roy inequality by a factor of $2^{(M-1)}$ (M is the number of particles). The assumption to derive the Roy inequality is that the system is in separable states. The Roy inequality is derived by quadratic entanglement witness inequality presented in Ref. [21]. The GHZ state violates standard Bell inequalities [22–27] by a factor of $2^{(M-1)/2}$ [23, 24]. Roy concludes that the Roy inequality exponentially stronger than standard Bell inequalities by a factor of $2^{(M-1)/2}$. The Roy inequality is important since it reveals the crucial difference between the notion of quantum separability and time invariant rotationally variant local realistic theory. Time invariant rotationally variant local realistic theory violates the Roy inequality whereas all separable states satisfy the Roy inequality (see also Refs. [28–30]).

A fundamental question is addressed in conjunction with the difference between quantum theoretical correlations and realistic theoretical correlations in a finite-dimensional space. Can multiqubit pure uncorrelated state violate a time invariant nonlocal realistic theory? This question is not answered by the reports of the experiments [6–8] mentioned above. We show that multi spin-$1/2$ pure uncorrelated state violates a time invariant nonlocal realistic theory ($2N$-number particles with $N = 1, 2, \ldots$). $2N$ implies that we consider Bose-Einstein statistics. In what follows, we restrict ourselves to a physical system with a finite-dimensional space.

In this paper, we assume that time invariance of physical laws is true. We assume that one source of $2N$ uncorrelated spin-carrying particles emits them in a state, which can be described as a multipartite pure uncorrelated state ($+\infty > N \geq 1$). We assume that each of them is a spin-$1/2$ pure state lying in the $\frac{z+x}{\sqrt{2}}$ direction. We show that $2N$-particle pure uncorrelated quantum state violates a time invariant nonlocal realistic theory.

Our paper is important in the following reason. There is much research in which multi spin-1/2 pure uncorrelated states are applied to implement quantum computation. We see that multi spin-1/2 pure uncorrelated state violates a time invariant nonlocal realistic theory. The implementation of quantum computation by using multi spin-$1/2$ pure uncorrelated state a state rules out a time invariant nonlocal realistic theory.

2 Multipartite Pure Uncorrelated State Violates Nonlocal Realism

Assume that we have a set of $2N$ spins $\frac{1}{2}$. Each of them is a spin-$1/2$ pure state lying in the $\frac{z+x}{\sqrt{2}}$ direction. Assume that one source of $2N$ uncorrelated spin-carrying particles emits them in a state, which can be described as a multi spin-$1/2$ pure uncorrelated state. Parameterize the settings of the jth observer with a unit vector $\vec{n}_j$ (its direction along which the spin component is measured) with $j = 1, \ldots, 2N$. One can introduce the 'nonlocal' correlation function, which is the average of the product of the nonlocal results

$$E_{\mathrm{NLR}}(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_{2N}) = \langle r(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_{2N})\rangle_{\mathrm{avg}} \tag{1}$$

where r is the nonlocal result. Assume the value of r is ± 1 (in $(\hbar/2)^{2N}$ unit), which is obtained if the measurement directions are set at $\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_{2N}$.

One can introduce a quantum correlation function with the system in multi spin-$1/2$ pure uncorrelated state

$$E_{\mathrm{Sep}}(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_{2N}) = \mathrm{tr}[\rho \vec{n}_1 \cdot \vec{\sigma} \otimes \vec{n}_2 \cdot \vec{\sigma} \otimes \cdots \otimes \vec{n}_{2N} \cdot \vec{\sigma}] \tag{2}$$

where $\otimes$ is the tensor product, $\cdot$ is the scalar product in $\mathbf{R}^2$, $\vec{\sigma} = (\sigma_z, \sigma_x)$ is a vector of two Pauli operators, and ρ is multi spin-$1/2$ pure uncorrelated state

$$\rho = \rho_1 \otimes \rho_2 \otimes \cdots \otimes \rho_{2N} \tag{3}$$

with $\rho_j = |\Psi_j\rangle\langle\Psi_j|$ and $|\Psi_j\rangle$ is a spin-$1/2$ pure state lying in the $\frac{z+x}{\sqrt{2}}$ direction.

One can introduce the observable (unit) vector $\vec{n}_j$ in a plane coordinate system as follows

$$\vec{n}_j(\theta_j^{k_j}) = \cos\theta_j^{k_j} \vec{x}_j^{(1)} + \sin\theta_j^{k_j} \vec{x}_j^{(2)}, \tag{4}$$

where $\vec{x}_j^{(1)} = \vec{z}$ and $\vec{x}_j^{(2)} = \vec{x}$ are the Cartesian axes. The angle $\theta_j^{k_j}$ takes two values (two-setting model [9])

$$\theta_j^1 = 0 \quad \text{and} \quad \theta_j^2 = \frac{\pi}{2}. \tag{5}$$

We derive a necessary condition to be satisfied by the quantum correlation function with the system in multi spin-$1/2$ pure uncorrelated state given in (2). In more detail, we derive the value of the scalar product of the quantum correlation function, E_{Sep} given in (2), i.e., $(E_{\mathrm{Sep}}, E_{\mathrm{Sep}})$. We use decomposition (4). We introduce simplified notations as

$$T_{i_1, i_2, \ldots, i_{2N}} = \mathrm{tr}[\rho \vec{x}_1^{(i_1)} \cdot \vec{\sigma} \otimes \vec{x}_2^{(i_2)} \cdot \vec{\sigma} \otimes \cdots \otimes \vec{x}_{2N}^{(i_{2N})} \cdot \vec{\sigma}] \tag{6}$$

and

$$\vec{c}_j = (c_j^1, c_j^2) = (\cos\theta_j^{k_j}, \sin\theta_j^{k_j}). \tag{7}$$

Then, we have

$$(E_{\mathrm{Sep}}, E_{\mathrm{Sep}}) = \sum_{k_1=1}^{2} \cdots \sum_{k_{2N}=1}^{2} \left(\sum_{i_1, \ldots, i_{2N}=1}^{2} T_{i_1, \ldots, i_{2N}} c_1^{i_1} \cdots c_{2N}^{i_{2N}} \right)^2 = \sum_{i_1, \ldots, i_{2N}=1}^{2} T_{i_1, \ldots, i_{2N}}^2 \leq 1, \tag{8}$$

where we use the orthogonality relation $\sum_{k_j=1}^{2} c_j^{\alpha} c_j^{\beta} = \delta_{\alpha,\beta}$. The value of $\sum_{i_1,\ldots,i_{2N}=1}^{2} T_{i_1,\ldots,i_{2N}}^2$ is bounded as $\sum_{i_1,\ldots,i_{2N}=1}^{2} T_{i_1,\ldots,i_{2N}}^2 \leq 1$. We have

$$\prod_{j=1}^{2N} \sum_{i_j=1}^{2} (\mathrm{tr}[\rho_j \vec{x}_j^{(i_j)} \cdot \vec{\sigma}])^2 \leq 1. \tag{9}$$

From the convex argument, all separable states must satisfy the inequality (8). It is an inequality. It is important that the inequality (8) is saturated if ρ is multi spin-1/2 pure uncorrelated state such that, for every j, $|\Psi_j\rangle$ is a spin-1/2 pure state lying in the $\frac{z+x}{\sqrt{2}}$ direction. The reason of the inequality (8) is due to the following quantum inequality

$$\sum_{i_j=1}^{2} (\mathrm{tr}[\rho_j \vec{x}_j^{(i_j)} \cdot \vec{\sigma}])^2 \leq 1. \tag{10}$$

The inequality (10) is saturated if $\rho_j = |\Psi_j\rangle\langle\Psi_j|$ and $|\Psi_j\rangle$ is a spin-1/2 pure state lying in the $\frac{z+x}{\sqrt{2}}$ direction. The inequality (8) is saturated iff the inequality (10) is saturated for every j. Thus we have the maximal possible value of the scalar product as a quantum proposition

$$(E_{\mathrm{Sep}}, E_{\mathrm{Sep}})_{\max} = 1 \tag{11}$$

when the system is in multi spin-1/2 pure uncorrelated state.

A correlation function satisfies nonlocal realistic theories if it can be written as

$$E_{\mathrm{NLR}}(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_{2N}) = \sum_{\lambda} \rho(\lambda) I(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_{2N}, \lambda), \tag{12}$$

where λ is a set of hidden variables, $\rho(\lambda)$ is probability distribution, and I is random varianble which denotes a result of the measurement of the observables parameterized by directions of $\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_{2N}$.

Assume the quantum correlation function with the system in multi spin-1/2 pure uncorrelated state given in (2) admits a time invariant nonlocal realistic theory. One has the following proposition concerning a time invariant nonlocal realistic theory

$$E_{\mathrm{Sep}}(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_{2N}) = \sum_{\lambda_t} \rho(\lambda_t) I(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_{2N}, \lambda_t). \tag{13}$$

The subscript t expresses that a set of hidden variables λ_t is determined uniquely when a time t is specified.

In what follows, we show that we cannot assign the truth value "1" for the proposition (13) concerning a time invariant nonlocal realistic theory.

Assume the proposition (13) is true.

By changing the label λ_t into λ_{t_1}, we have same proposition

$$E_{\mathrm{Sep}}(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_{2N}) = \sum_{\lambda_{t_1}} \rho(\lambda_{t_1}) I(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_{2N}, \lambda_{t_1}). \tag{14}$$

By changing the label λ_t into λ_{t_2}, we have same proposition

$$E_{\mathrm{Sep}}(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_{2N}) = \sum_{\lambda_{t_2}} \rho(\lambda_{t_2}) I(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_{2N}, \lambda_{t_2}). \tag{15}$$

An important note here is that the value of the right-hand-side of (14) is equal to the value of the right-hand-side of (15) because we only change the label.

Hence one has

$$
\begin{aligned}
&(E_{\text{Sep}}, E_{\text{Sep}}) \\
&= \sum_{k_1=1}^{2} \cdots \sum_{k_{2N}=1}^{2} \sum_{\lambda_{t_1}} \rho(\lambda_{t_1}) I(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_{2N}, \lambda_{t_1}) \times \sum_{\lambda_{t_2}} \rho(\lambda_{t_2}) I(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_{2N}, \lambda_{t_2}) \\
&= \sum_{k_1=1}^{2} \cdots \sum_{k_{2N}=1}^{2} \sum_{\lambda_{t_1}} \rho(\lambda_{t_1}) \sum_{\lambda_{t_2}} \rho(\lambda_{t_2}) I(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_{2N}, \lambda_{t_1}) \times I(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_{2N}, \lambda_{t_2}) \\
&\leq \sum_{k_1=1}^{2} \cdots \sum_{k_{2N}=1}^{2} \sum_{\lambda_{t_1}} \rho(\lambda_{t_1}) \sum_{\lambda_{t_2}} \rho(\lambda_{t_2}) |I(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_{2N}, \lambda_{t_1}) \times I(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_{2N}, \lambda_{t_2})| \\
&= \sum_{k_1=1}^{2} \cdots \sum_{k_{2N}=1}^{2} \sum_{\lambda_{t_1}} \rho(\lambda_{t_1}) \sum_{\lambda_{t_2}} \rho(\lambda_{t_2}) = 2^{2N}. \qquad (16)
\end{aligned}
$$

We use the following fact

$$|I(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_{2N}, \lambda_{t_1}) \times I(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_{2N}, \lambda_{t_2})| = +1. \qquad (17)$$

Time invariance of physical laws says that the inequality (16) is saturated since

$$
\begin{aligned}
&\{\lambda_{t_1} | I(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_{2N}, \lambda_{t_1}) = 1\} = \{\lambda_{t_2} | I(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_{2N}, \lambda_{t_2}) = 1\} \quad \text{and} \\
&\{\lambda_{t_1} | I(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_{2N}, \lambda_{t_1}) = -1\} = \{\lambda_{t_2} | I(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_{2N}, \lambda_{t_2}) = -1\}.
\end{aligned} \qquad (18)
$$

Thus we have the following proposition concerning a time invariant nonlocal realistic theory

$$(E_{\text{Sep}}, E_{\text{Sep}})_{\max} = 2^{2N}. \qquad (19)$$

We cannot assign the truth value "1" for two propositions (11) (concerning a quantum proposition (9)) and (19) (concerning a time invariant nonlocal realistic theory), simultaneously, when the system is in multi spin-$1/2$ pure uncorrelated state. Each of them is a spin-$1/2$ pure state lying in the $\frac{z+x}{\sqrt{2}}$ direction. We are in the contradiction when the system is multi spin-$1/2$ pure uncorrelated state. We cannot accept the validity of the proposition (13) (concerning a time invariant nonlocal realistic theory) if we assign the truth value "1" for a quantum proposition (11). Multi spin-$1/2$ pure uncorrelated state violates a time invariant nonlocal realistic theory.

3 Conclusions

In conclusion, we have assumed that time invariance of physical laws is true. We have assumed that one source of $2N$ uncorrelated spin-carrying particles emits them in a state, which can be described as a multipartite pure uncorrelated state ($+\infty > N \geq 1$). We have assumed that each of them is a spin-$1/2$ pure state lying in the $\frac{z+x}{\sqrt{2}}$ direction. We have assumed that the measurement setup is two-orthogonal-settings for each of the observers. We have shown that $2N$-particle pure uncorrelated quantum state violates a time invariant nonlocal realistic theory. $2N$ implies that we consider Bose-Einstein statistics.

References

1. Peres, A.: Quantum Theory: Concepts and Methods. Kluwer, Dordrecht (1993)
2. Redhead, M.: Incompleteness, Nonlocality, and Realism, 2nd edn. Clarendon, Oxford (1989)
3. Einstein, A., Podolsky, B., Rosen, N.: Phys. Rev. **47**, 777 (1935)
4. Bell, J.S.: Physics **1**, 195 (1964)
5. Leggett, A.J.: Found. Phys. **33**, 1469 (2003)
6. Gröblacher, S., Paterek, T., Kaltenbaek, R., Brukner, Č., Żukowski, M., Aspelmeyer, M., Zeilinger, A.: Nature **446**, 871 (2007)
7. Paterek, T., Fedrizzi, A., Gröblacher, S., Jennewein, T., Żukowski, M., Aspelmeyer, M., Zeilinger, A.: Phys. Rev. Lett. **99**, 210406 (2007)
8. Branciard, C., Ling, A., Gisin, N., Kurtsiefer, C., Lamas-Linares, A., Scarani, V.: Phys. Rev. Lett. **99**, 210407 (2007)
9. Nagata, K.: J. Phys. A: Math. Theor. **41**, 155308 (2008)
10. de Oliveira, A.N., Walborn, S.P., Monken, C.H.: J. Opt. B: Quantum Semiclass. Opt. **7**, 288–292 (2005)
11. Kim, Y.-H.: Phys. Rev. A **67**, 040301(R) (2003)
12. Mohseni, M., Lundeen, J.S., Resch, K.J., Steinberg, A.M.: Phys. Rev. Lett. **91**, 187903 (2003)
13. Tame, M.S., Prevedel, R., Paternostro, M., Böhi, P., Kim, M.S., Zeilinger, A.: Phys. Rev. Lett. **98**, 140501 (2007)
14. Nielsen, M.A., Chuang, I.L.: Quantum Computation and Quantum Information. Cambridge University Press, Cambridge (2000)
15. Galindo, A., Martín-Delgado, M.A.: Rev. Mod. Phys. **74**, 347 (2002)
16. Werner, R.F.: Phys. Rev. A **40**, 4277 (1989)
17. Malley, J.D.: Phys. Rev. A **69**, 022118 (2004)
18. Nagata, K.: Phys. Rev. A **73**, 066101 (2006)
19. Roy, S.M.: Phys. Rev. Lett. **94**, 010402 (2005)
20. Greenberger, D.M., Horne, M.A., Zeilinger, A.: In: Kafatos, M. (ed.) Bell's Theorem, Quantum Theory and Conceptions of the Universe, pp. 69–72. Kluwer, Dordrecht (1989)
21. Nagata, K., Koashi, M., Imoto, N.: Phys. Rev. Lett. **89**, 260401 (2002)
22. Żukowski, M., Brukner, Č.: Phys. Rev. Lett. **88**, 210401 (2002)
23. Werner, R.F., Wolf, M.M.: Phys. Rev. A **64**, 032112 (2001)
24. Werner, R.F., Wolf, M.M.: Quantum Inf. Compt. **1**, 1 (2001)
25. Mermin, N.D.: Phys. Rev. Lett. **65**, 1838 (1990)
26. Roy, S.M., Singh, V.: Phys. Rev. Lett. **67**, 2761 (1991)
27. Belinskii, A.V., Klyshko, D.N.: Phys. Usp. **36**, 653 (1993)
28. Nagata, K.: Phys. Rev. A **76**, 024101 (2007)
29. Seevinck, M., Uffink, J.: Phys. Rev. A **76**, 042105 (2007)
30. Uffink, J., Seevinck, M.: Phys. Lett. A **372**, 1205 (2008)